ALGEBRA.

433. Proposed by B. J. BROWN, student at Drury College.

Prove that, if all the quantities, a, b, etc., are real, then all the roots of the equations

$$\begin{vmatrix} a-x & h \\ h & b-x \end{vmatrix} = 0, \qquad \begin{vmatrix} a-x & h & g \\ h & b-x & f \\ g & f & c-x \end{vmatrix} = 0$$

are real; and generalize the proposition.

434. Proposed by S. A. JOFFE, New York City.

Express the "difference of zero," $\Delta^n 0^{n+1}$, in the form, $C_1(n+2)! - C_2(n+1)!$, where C_1 and C_2 are numerical coefficients independent of n.

435. Proposed by C. N. SCHMALL, New York City.

Show that $(e-1) - \frac{1}{2}(e-1)^2 + \frac{1}{3}(e-1)^3 - \cdots = 1$, where e is the Naperian base of logarithms.

GEOMETRY.

463. Proposed by B. J. BROWN, student at Drury College.

If μ and ν are the parameters of the two confocal conics through any point on the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

show that $\mu + \nu + a^2 + c^2 = 0$, along a central circular section.

464. Proposed by FRANK R. MORRIS, Glendale, Calif.

The sum of the hypotenuse and one side of a right triangle is 100 feet. A point on the hypotenuse is 10 feet from each of the sides. Find the length of the hypotenuse correct to the third decimal place.

465. Proposed by ROGER A. JOHNSON, Western Reserve University.

Let C be a fixed circle, A a point outside it. Let AT and AT' be the tangents from A to the circle, touching the latter at T and T'. Let two secants be drawn through A, cutting the circle at P, Q and R, S respectively. Let PR and QS meet at X, PS and QR meet at Y. Prove by elementary methods that for all positions of the secants, X and Y lie on the line TT'.

CALCULUS.

383. Proposed by WILLIAM CULLUM, Albion, Mich.

Find the area of the curved surface of a right cone whose base is the asteroid, $x^{2/3} + y^{2/3} = a^{2/3}$, and whose altitude is h.

From Townsend and Goodenough's *First Course in Calculus*, p. 288, Ex. 11.

Note.—Among other methods, find the required area by means of the formula

$$\int \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dxdy.$$

EDITORS.

384. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

In what time will a sum of money double itself at 6 per cent. interest per annum if compounded at indefinitely short intervals?

385. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $f(x)$ is continuous between a and x, show that

$$\lim_{n \doteq \infty} \frac{n!}{(x-a)^n} \int_a^x \cdots \int_a^x f(x) dx^n = f(a).$$

386. Proposed by HERBERT N. CARLETON, Newberry, Mass.

C is a fixed point on the perpendicular bisector of the line segment AB. Locate a point D also on this bisector, such that $AD + BD + DC$ shall be a minimum.

MECHANICS.

309. Proposed by JOS. B. REYNOLDS, Lehigh University.

The tangent at one cusp of a vertical, three arched hypocycloid is horizontal, and a particle will just slide under gravity from the upper cusp to this cusp. Find the equation which the coefficient of friction must satisfy.

310. Proposed by EMMA M. GIBSON, Drury College.

A particle movable on a smooth spherical surface of radius a is projected along a horizontal great circle with a velocity v which is great compared with $\sqrt{2ga}$. Prove that its path lies between this great circle and a parallel circle whose plane is approximately at a depth $2ga^2/v^2$ below the center.

From Lamb's *Dynamics*, p. 334, Ex. 3.

SOLUTIONS OF PROBLEMS.

Note.—When several persons send in solutions for the same problem, the committee naturally and, we think, properly select for publication that one which is not only correct mathematically but is written out in the best form for publication. They must either do this or else, if they select solutions which are in poor form, in order to give as many solvers as possible a chance, they must write these solutions over to save them from rejection by the Managing Editor as bad copy. The task of *putting solutions into acceptable shape for the printer* is one which the members of the committee do not relish,—and who can blame them? This will explain why some names appear more frequently at the head of the solutions than do others, even though several may have solved the same problem. See the suggestions for preparing solutions published in several previous issues. MANAGING EDITOR.

ALGEBRA.

409. Proposed by C. E. GITHENS, Wheeling, W. Va.

Find integral values for the edges of a rectangular parallelopiped so that its diagnoal shall be rational.

REMARKS BY W. C. EELLS, U. S. Naval Academy.

In the February, 1915, issue of the MONTHLY (pp. 60–61) Artemas Martin criticizes my solution of this problem in the October, 1914, issue, stating that I have solved a different problem from the one proposed, and that I claimed that a certain rational parallelopiped was the smallest possible one, whereas he exhibits four others that are smaller.

Since the problem reads "Find integral values" and not "Find *all* integral values, etc." it was in order to impose the condition $x^2 + y^2 = k^2$ or any other condition so long as integral solutions of the equation $x^2 + y^2 + z^2 = a^2$ were found. I showed two general methods of solution, each giving an infinite number of *prime* integral solutions, but did not state nor even suppose that I had found all possible solutions. I fail to see, however, how I solved a *different* problem from the one proposed.

Under my first method, as an example, I gave the solution $(x, y, z, a) = (4, 3, 12, 13)$ as the smallest rational parallelopiped, and it should have been sufficiently